## REPORT No. 572

## DETERMINATION OF THE CHARACTERISTICS OF TAPERED WINGS

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#### SUMMARY

Tables and charts for use in determining the characteristics of tapered wings are presented. Theoretical factors are given from which the following characteristics of tapered wings may be found: The span lift distribution, the induced-angle-of-attack distribution, the lift-curve slope, the angle of zero lift, the induced drag, the aero-dynamic-center position, and the pitching moment about the aerodynamic center.

The wings considered cover the complete range of taper ratios and a range of aspect ratios from 2 to 20. The factors given include the effects of sweepback and twist and apply to wings having a straight taper plan form with rounded tips and an elliptical plan form. The general formulas of the usual wing theory are also given from which the characteristics of a wing of any form may be calculated when the section characteristics are known from experiment.

In addition to the tables and charts, test results are given for nine tapered wings, including wings with sweep-back and twist. The test results verify the values computed by the methods presented in the first part of the report. A final section is given outlining a method for estimating the lift coefficient at which a tapered wing begins to stall. This method, which should be useful for estimating the maximum lift coefficient of tapered wings, is applied to one of the wings tested.

### INTRODUCTION

A large amount of work has been done on the determination of tapered-wing characteristics from airfoil theory. Glauert has given some of the characteristics of wings with straight taper for a limited range of aspect ratios (references 1 and 2). Hueber has given other characteristics of wings with straight taper for a large range of aspect ratios (reference 3). Several other papers have given various characteristics of tapered wings. The data of all the papers, however, have been limited by one or more of the following factors: Range of aspect ratio and taper ratio, number of characteristics given, and omission of data on wings with sweepback and twist. In order to provide more complete information, data are given in this report for a large range of aspect ratios, for the complete range

of taper ratios, and for wings with sweepback and twist. As airplane wings are usually rounded at the tips, the data are given for wings with rounded tips.

In addition to the theoretical characteristics, the results of tests of nine tapered wings, including wings with sweepback and twist, and a comparison of some of the test results with theoretical values are presented.

The characteristics are given for wings having a straight taper and rounded tips and for wings having an elliptical plan form, with an aspect-ratio range from 2 to 20. For these wings, formulas are given using factors that are presented in tables and charts. From the formulas and factors the following characteristics of tapered wings may be determined: Span lift distribution, induced-angle-of-attack distribution, lift-curve slope, angle of zero lift, induced drag, aerodynamic-center position, and pitching moment about the aerodynamic center.

# METHOD OF OBTAINING DATA BASIC CONCEPTS

When obtaining the data used to determine the characteristics of wings, a tapered wing is considered to consist of a series of airfoil sections that may vary in shape, chord length, and in angle of attack from root to tip. Each airfoil section is assumed to have an aerodynamic center through which the lift and drag act and about which the pitching moment is constant.

With the section characteristics as a basis, characteristics of the entire wing are obtained by integration across the span. Formulas for the integrations will first be given for a wing of any shape and zero dihedral; that is, the aerodynamic centers of all the sections along the span lie in a plane which passes through the root chord and which is perpendicular to the plane of symmetry. Wings of particular shape will be considered later and a method for including the effect of dihedral will be given.

For any tapered wing the span lift distribution may be considered to consist of two parts. One part, which will be called the "basic distribution," is the distribution that depends principally on the twist of the wing and occurs when the total lift of the wing is zero; it does not change with the angle of attack of the wing. The second part of the span lift distribution, which will be called the "additional distribution," is the lift due to change of the wing angle of attack; it is independent of the wing twist and maintains the same form throughout the reasonably straight part of the lift curve.

In the designation of the characteristics of a wing, lower-case letters will be used for section characteristics and upper-case letters for the characteristics of the entire wing. The basic and additional section lift coefficients are then  $c_{l_b}$  and  $c_{l_a}$ . A complete list of symbols follows. It is convenient to find the additional lift coefficient for a wing  $C_L$  of 1 and it is then designated  $c_{l_{a1}}$ . The two coefficients are related by  $c_{l_a} = C_L c_{l_{a1}}$ . The total lift coefficient at any section is found from the basic and additional coefficients from

$$c_{l_0} = c_{l_b} + C_L c_{l_{a1}}$$

where  $c_{l_0}$  is the lift coefficient perpendicular to the local relative wind at any section as distinguished from  $c_i$ , which is perpendicular to the relative wind at a distance. For convenience, however,  $c_i$  will be used and may be considered equal to  $c_{l_0}$ .

#### SYMBOLS

A, aspect ratio,  $b^2/S$ .

b, span.

c, chord at any section along the span.

c<sub>i</sub>, tip chord (for rounded tips, c<sub>i</sub> is the fictitious chord obtained by extending the leading and trailing edges to the extreme tip).

c, chord at root of wing or plane of symmetry.

S, wing area.

β, angle of sweepback, measured between the lateral axis and a line through the aerodynamic centers of the wing sections. (See fig. 1.)

ϵ, aerodynamic twist in degrees from root to tip, measured between the zero-lift directions of the center and tip sections, positive for washin.

x, longitudinal coordinate, parallel to the root chord.

y, lateral coordinate, perpendicular to plane of symmetry.

z, vertical coordinate in the plane of symmetry, perpendicular to the root chord.

 $x_{a.c.}$ , x coordinate of wing aerodynamic center.

a, wing lift-curve slope, per degree.

 $a_0$ , wing section lift-curve slope, per degree.

m, wing lift-curve slope, per radian.

 $m_0$ , wing section lift-curve slope, per radian.

 $\alpha$ , angle of attack at any section along the span.

 $\alpha_s$ , wing angle of attack measured from the chord of the root section.

 $\alpha_{a_s}$ , absolute wing angle of attack measured from the zero-lift direction of the root section.

 $\alpha_{l_0}$ , angle of zero lift of the root section.

 $\alpha_{l_0}$ , angle of zero lift of the tip section.

 $\alpha_{s_{(L=0)}}$ , wing angle of attack for zero lift.

 $\alpha_i$ , section induced angle of attack.

c<sub>l</sub>, section lift coefficient perpendicular to the distant relative wind.

Subscripts for  $c_i$ :

 refers to section lift coefficient perpendicular to the local relative wind.

b, refers to basic lift  $(C_L=0)$ .

a, refers to additional lift (any  $C_L$ ).

a1, refers to additional lift  $(C_L=1)$ .

 $c_{dp}$  section induced-drag coefficient.

 $c_{d_0}$ , section profile-drag coefficient.

 $c_{m_{a.e.}}$ , section pitching-moment coefficient about section aerodynamic center.

l, section lift.

 $m_{l_a}$ , section pitching moment due to additional lift forces.

 $M_{l_a}$ , wing pitching moment due to additional lift forces.

 $C_{m_{l_a}}$ , wing pitching-moment coefficient due to additional lift forces.

 $C_{m_{I_b}}$ , wing pitching-moment coefficient due to basic lift forces.

 $C_{m_S}$ , wing pitching-moment coefficient due to the pitching moments of the wing sections.

 $C_{m_{a.e.}}$ , wing pitching-moment coefficient about its aerodynamic center.

 $C_L$ , wing lift coefficient.

 $C_{D,p}$  wing induced-drag coefficient.

## GENERAL FORMULAS

Formulas in terms of the section characteristics.— The induced angle of attack at any section is obtained from  $c_i$  by

$$\alpha_i = \alpha - \frac{c_i}{m_0}$$

The section induced-drag coefficient is obtained from  $\alpha_i$  and  $c_i$  from

$$c_{d_i} = \alpha_i c_i$$

and the induced-drag coefficient for the entire wing may be obtained by integration across the semispan from the section values:

$$C_{D_i} = \frac{2}{S} \int_0^{b/2} \alpha_i c_i c dy \tag{1}$$

In order to obtain the aerodynamic center and the pitching moment of the wings, a system of reference axes was used; the origin was at the aerodynamic center of the root section and the axes were as shown in figure 1. The x axis (fig. 1 (a)) is parallel to the root chord, and the y axis (fig. 1 (b)) is perpendicular to the plane of symmetry with positive directions following the vectors. The wing axis is the locus of the aerodynamic centers of the sections and lies in the x-y plane. The lift l and the coefficient  $c_l$  of any section along the span are represented in figure 1.

located at a distance x from the y axis has a moment arm of

and a pitching moment about the lateral axis (fig. 1) due to the additional lift force of

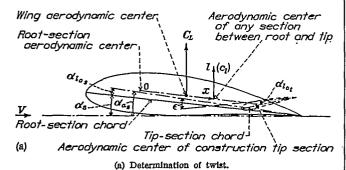
$$m_l = -x \cos \alpha_l l_a$$

but the lift increment of any section is

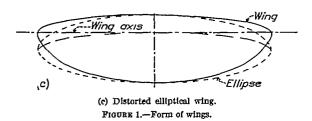
$$l_a = c_{l_a} qc$$

and the pitching moment for the entire wing is obtained from

$$M_{l_{\bullet}} = -2q \cos \alpha_{\bullet} \int_{0}^{b/2} c_{l_{\bullet}} cxdy$$



Root-section aerodynamic center riwing axis Construction tip section Wing aerodynomic center (b) Straight-taper wing with rounded tips.



Pitching-moment coefficients for the entire wing will be based on a chord length of S/b so that

$$C_m = \frac{Mb}{qS^2}$$

The pitching-moment coefficient due to the additional lift forces then becomes

$$C_{m_l} = -\frac{2b}{S^2} \cos \alpha_s \int_0^{b/2} c_l \, cxdy$$

The additional lift forces have a centroid through which the lift may be considered to act. This point is the aerodynamic center of the wing and its x coordi-

A typical section with the aerodynamic center (nate will be designated  $x_{\sigma.c.}$ . (See fig. 1.) This distance corresponds to d in reference 4. The term  $C_{m_l}$  then may also be expressed

$$C_{m_{l_a}} = -(x_{a.c.} \cos \alpha_s) \frac{b}{\overline{S}} C_L$$

If the previous expression for  $C_{m_l}$  is used,  $x_{a.c.}$  is obtained as a fraction of S/b by

$$\frac{x_{a.c.}}{S/b} = \frac{\frac{2b}{S^2} \int_0^{b/2} c_l cxdy}{C_{r.}}$$
 (2)

The moment due to the drag forces has been omitted because it is relatively small, except for wings with large amounts of sweepback or dihedral.

The pitching moment of the basic lift forces is a couple and is therefore independent of the axis about which it is determined. The lateral axis was used to facilitate computation but, when the pitching moment is used, it is convenient to consider it constant about an axis through the aerodynamic center. According to the method previously used, the pitching-moment coefficient due to the basic lift forces is

$$C_{m_{l_{i}}} = \pm \frac{2}{\overline{S^{2}}} b \int_{0}^{b/2} c_{l_{i}} cx dy \tag{3}$$

The cos  $\alpha_{\epsilon_{(L=0)}}$  (the cosine of the angle of zero lift of the wing measured from the root chord) has been omitted because it is practically equal to unity.

In addition to the basic lift forces, the pitching moment of each section also contributes to the pitching moment of the wing, which is obtained by

$$C_{m_s} = \frac{2b}{S^2} \int_0^{b/2} c_{m_{a.c.}} c^2 dy \tag{4}$$

The total moment about the aerodynamic center is then the sum of the two foregoing parts

$$C_{m_{g,c}} = C_{m_l} + C_{m_g}$$

Formulas in terms of the coefficients of the Fourier series.—In order to obtain data from the foregoing formulas, the spanwise distribution of the lift coefficient (following Glauert) was expressed as the Fourier series:

$$c_{l} = \frac{4b}{c} \sum A_{n} \sin n\theta$$

where  $\theta$  is related to the distance along the span (fig. 1) by  $y=(-b/2)\cos\theta$  and only odd values of n are used. When  $c_i$  is expressed in the foregoing manner, it is possible to obtain the induced angle of attack in the form

$$\alpha_i = \sum n A_n \frac{\sin n\theta}{\sin \theta}$$

Also the coefficients  $A_{\pi}$  may be expressed in the form

$$A_n = B_n \alpha_a + C_n \epsilon$$

where  $\alpha_{e_i}$  is the absolute angle of attack of the root section; that is, the angle of attack of the root section, measured from its direction of zero lift, and  $\epsilon$  is the wing twist measured between the zero-lift directions of the root and tip sections.

When the preceding expressions for  $c_i$  and  $\alpha_i$  are substituted in the foregoing formulas, the characteristics are obtained in terms of the coefficients  $B_n$  and  $C_n$ , which in turn are grouped into factors.

From (1) the induced-drag coefficient may be obtained in the form:

$$C_{D_i} = \frac{C_L^2}{\pi A u} + C_L \epsilon a_0 v + (\epsilon a_0)^2 w$$

where A is the aspect ratio, and

$$\frac{1}{u} = \frac{1}{B_1^2} \left[ \sum_{\substack{n=3, 5, 7 \\ \dots \dots \infty}} nB_n^2 \right] + 1$$

$$v = \frac{2}{m_0 B_1} \left[ \sum_{\substack{n=3, 5, 7 \\ \dots \dots \infty}} nB_n \left( C_n - \frac{C_1}{B_1} B_n \right) \right]$$

$$w = \frac{\pi A}{m_0^2} \left[ \sum_{\substack{n=3, 5, 7 \\ \dots \dots \infty}} n \left( C_n - \frac{C_1}{B_1} B_n \right)^2 \right]$$

In the determination of the aerodynamic-center position, the wing axis is considered to be a straight line and the angle of sweepback is  $\beta$  (fig. 1), then

$$x = |y| \tan \beta$$

and from (2) the x coordinate of the aerodynamic center is obtained as

$$\frac{x_{a.c.}}{S/b} = HA \tan \beta$$

where

$$H = \frac{2}{\pi B_1} \left( \frac{B_1}{3} + \frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} + \dots \right)$$

$$\frac{B_n}{4} \left\{ \frac{\sin \left[ (n-2)\pi/2 \right]}{(n-2)} - \frac{\sin \left[ (n+2)\pi/2 \right]}{(n+2)} \right\}$$

From (3) the moment due to the basic lift forces becomes

$$C_{m_{I_b}} = -G\epsilon a_0 A \tan \beta$$

where  $a_0$  is the section lift-curve slope for the wing and

$$G = \frac{2A}{m_0} \left[ \left( \frac{C_3}{5} - \frac{C_5}{21} + \frac{C_7}{45} \dots \right) - \frac{C_1}{B_1} \left( \frac{B_3}{5} - \frac{B_5}{21} + \frac{B_7}{45} \dots \right) \right]$$

(The term  $C_{m_{I_h}}$  is equal to  $C_{m_T}$  in reference 4.)

Also from equation (4) the pitching moment of the wing due to the pitching moments of the sections is expressed as

$$C_{m_{\bullet}} = Ec_{m_{\sigma}}$$

where  $c_{m_{\alpha}}$  is constant across the span and

$$E = \frac{2b}{S^2} \int_0^{b/2} c^2 dy$$

In addition to the foregoing formulas, the following by distorting an ellipse until the wing axis becomes formulas were obtained in terms of  $B_n$  and  $C_n$  for other straight. In order to determine the wing axis, the

characteristics. The basic and additional lifts at any point along the span were expressed by the dimensionless quantities

$$L_b = \frac{4A}{m_0} \left[ \sum_{n=3} \left( C_n - \frac{C_1}{B_1} B_n \right) \sin n\theta \right]$$

and

$$L_a = \frac{4}{\pi} \left[ \sum_{n=1,3,5,7} \frac{B_n}{B_1} \sin n\theta \right]$$

so that

$$c_{l_b} = \frac{\epsilon a_0 S}{cb} L_b$$

and

$$c_{l_{a1}} = \frac{S}{ch} L_a$$

The lift-curve slope was obtained in the form

$$a = \frac{\pi A B_1}{57.3}$$

By the introduction of the slope for an elliptical wing, a may be expressed

$$a = f \frac{a_0}{1 + \frac{57.3a_0}{\pi A}}$$

where

$$f = \frac{a}{a_0} \left( 1 + \frac{57.3a_0}{\pi A} \right)$$

The angle of zero lift was obtained in the form

$$\frac{\alpha_{a_s}}{\epsilon} = -\frac{C_1}{B_1} = J$$

The angle of attack of a wing may then by given by

$$\alpha_{s} = \frac{C_{L}}{a} + \alpha_{l_{0s}} + J\epsilon$$

where  $\alpha_*$  is the angle of attack measured from the chord of the root section, and  $\alpha_{l_{0_*}}$  is the angle of zero lift of the root section.

The general formulas and the factors used with them have now been outlined. The manner of obtaining the data will be completed by explaining the method of finding the coefficients  $B_n$  and  $C_n$  used in computing the factors.

Determination of the coefficients of the Fourier series.—The coefficients  $B_n$  and  $C_n$  depend on the shape of the wing. The two wing shapes used are shown on figure 1. Wing (b) has a straight taper plan form with rounded tips and (c) an elliptical plan form. The tapered wing is shown with sweepback and the elliptical wing without, but either wing may or may not have sweepback. The rounded tip of the tapered wing is formed within a trapezoidal tip of length  $c_i$ , and the taper of the wing is determined by the tip to root chord ratio  $c_i/c_i$ . The aerodynamic centers of the airfoil sections lie on a straight line across the semispan and form the wing axis. The elliptical wing is formed by distorting an ellipse until the wing axis becomes straight. In order to determine the wing axis, the

aerodynamic centers of the airfoil sections were taken at the quarter-chord point. The straight wing axis may then be given sweepback with each chord moving parallel to its original position. The same process would be used to change the sweepback of the tapered wing.

For the wings considered, the twist varies linearly from root to tip and the total angle of twist is  $\epsilon$ . As shown in figure 1,  $\epsilon$  is the twist measured between the zero-lift directions of the root and tip sections.

Tapered wing.—For the tapered wing the coefficients  $B_n$  and  $C_n$  were determined from the equation

$$\alpha_a = \sum A_n \sin n \, \theta \left( \frac{4b}{m_0 c} + \frac{n}{\sin \theta} \right) \tag{5}$$

where  $\dot{\alpha}_a$  is the absolute angle of attack at any section; that is, the angle of attack measured from the zero-lift direction for the section. The coefficients  $B_n$  and  $C_n$  are related to  $A_n$  by

$$A_n = B_n \alpha_{a_{\bullet}} + C_n \epsilon$$

where  $\alpha_{a_1}$  is the absolute angle of attack of the root section. The value of  $m_0$  used in the preceding equation was 5.79 per radian, which approximates the lift-curve slope of good airfoil sections. For the linear taper  $\alpha_a$  becomes

$$\alpha_a = \alpha_{a_s} + \epsilon \cos \theta$$

For a wing of any particular aspect ratio and taper ratio, equation  $_{\bullet}(5)$  was satisfied at four points along the semispan by the usual method (except for  $c_l/c_s=0$  for which six points were necessary to obtain sufficient accuracy), and values of  $B_n$  and  $C_n$  for n=1, 3, 5, and 7 were found.

The elliptical wing.—For the elliptical wing the foregoing fundamental equation may be simplified and a new series of coefficients, independent of aspect ratio, may be obtained. The coefficient  $A_n$  for n=3, 5, 7...  $\infty$  may be obtained in the form

$$A_{n} = \frac{k_{n}\epsilon}{\frac{\pi A}{m_{0}} + n}$$

where  $k_n$  is determined from

$$\cos \theta = k_3 \left( 1 + \frac{\sin 3\theta}{\sin \theta} \right) - k_5 \left( 1 - \frac{\sin 5\theta}{\sin \theta} \right) + k_7 \left( 1 + \frac{\sin 7\theta}{\sin \theta} \right) \cdot \cdot \cdot$$

The factors for the elliptical wing then take the form

$$L_{b}=4A\left[\sum_{\substack{n=3,5,7,\\\dots \infty}}\frac{k_{n}}{\pi A+nm_{0}}\sin n\theta\right]$$

$$L_{a}=\frac{4}{\pi}\sqrt{1-\left(\frac{y}{b/2}\right)^{2}}$$

$$a = \frac{a_0}{1 + \frac{57.3a_0}{\pi A}}$$

$$f = 1$$

$$J = -k_3 + k_5 - k_7 \dots$$

$$u = 1$$

$$v = 0$$

$$w = \frac{\pi A}{m_0^2} \left[ \sum_{n=3, 5, 7, \sqrt{\frac{\pi A}{m_0} + n}} \frac{nk_n^2}{\sqrt{\frac{\pi A}{m_0} + n}} \right]$$

$$H = \frac{2}{3\pi}$$

$$G = \frac{2k_3}{5\pi \left(1 + \frac{3m_0}{\pi A}\right)} - \frac{2k_5}{21\pi \left(1 + \frac{5m_0}{\pi A}\right)} + \frac{2k_7}{45\pi \left(1 + \frac{7m_0}{\pi A}\right)} \dots$$

$$E = \frac{32}{3\pi^2} (c_{m_a, c} \text{ constant along the span})$$

The foregoing factors were obtained for the elliptical wing and for a straight-taper wing with trapezoidal tips for a range of aspect ratios from 3 to 20 and of taper ratios from 0 to 1. The factors were also obtained for the tapered wing with rounded tips for a sufficient number of aspect ratios and taper ratios so that the complete range could be covered using the factors for the wing with trapezoidal tips as a guide. Cross plots were then made to obtain figures 2 to 9 and the values for wings with rounded tips presented in tables I and Although the factors become less reliable as the aspect ratio is decreased, it was considered desirable to extrapolate the curves to an aspect ratio of 2 as the factors in the low-aspect-ratio range may be of use in the absence of other data. Additional spanwise liftdistribution data computed for the elliptical wing are

## USE OF TABLES AND CHARTS

given in table III.

In order to find the characteristics of a wing having a straight taper and rounded tips or having an elliptical plan form, the tables and charts may be used directly.

The properties of the wing should first be determined; that is, the taper ratio  $c_t/c_s$ , aspect ratio A, span b, the area S, the aerodynamic twist  $\epsilon$  in degrees, the angle of sweepback  $\beta$ , and the average value of section lift-curve slope, as well as the section lift-curve slope  $a_0$ , the section pitching-moment coefficient  $c_{m_a}$ , and the chord c at convenient stations along the semispan.

The chord and  $a_0$  should be found at the spanwise stations given in tables I and II to facilitate finding the spanwise lift distribution. Then, for the values of  $c_t/c_s$  and A, values of  $L_b$  and  $L_a$  may be found from tables I and II by interpolation if necessary. The section lift coefficients  $c_{i_b}$  and  $c_{i_{a1}}$  are then found for each station along the semispan from

$$c_{l_b} = \frac{\epsilon a_0 S}{cb} L_b$$

$$c_{l_{a1}} = \frac{S}{cb} L_a$$

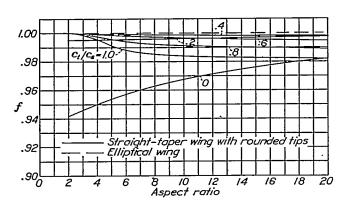


FIGURE 2.—Chart for determining lift-curve slope.

 $a = \int \frac{a_0}{1 + \frac{57.3 \ a_0}{\tau A}} \qquad m = 57.3a$ 

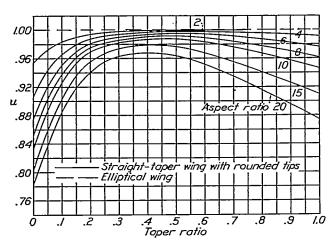


FIGURE 4.—Chart for determining induced-drag factor u.  $C_{Dl} = \frac{C_L{}^3}{\pi A u} + C_L{}^2 e a v + (e a_0){}^3 w$ 

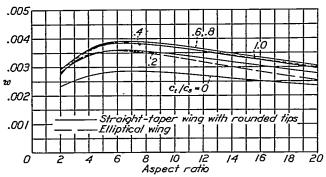


Figure 8.—Chart for determining induced-drag factor w.

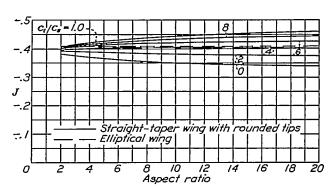


FIGURE 3.—Chart for determining angle of attack.

 $\alpha_{s} = \frac{C_{L}}{a} + \alpha_{l_{0}} + J_{4} \qquad \alpha_{s}_{(L=0)} = \alpha_{l_{0}} + J_{4}$ 

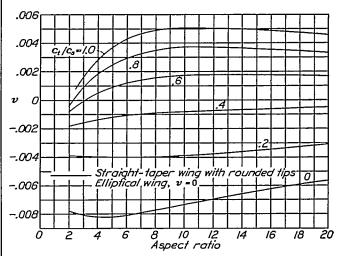


Figure 5.—Chart for determining induced-drag factor v.

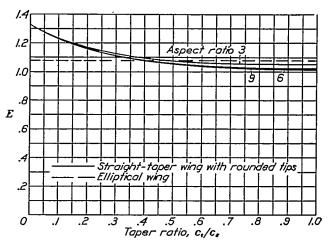


Figure 7.—Chart for determining pitching moment due to section moment.  $C_m = E c_{m_0,s_*}$ 

For  $c_{m_{a,a}}$  constant across the span.

and  $c_l$  for any value of  $C_L$  for the wing is obtained from



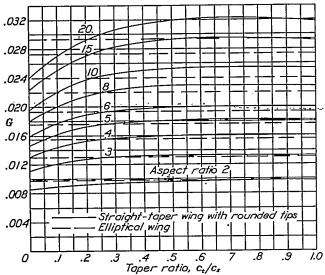


FIGURE 8.—Chart for determining pitching moment due to basic lift forces.  $C_{m_{I_k}} = -Gea_0 A \tan \beta$ 

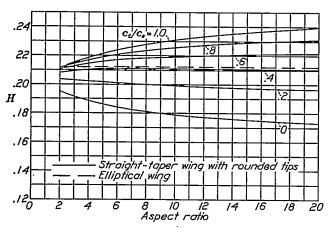


FIGURE 9.—Chart for determining aerodynamic-center position.

$$\frac{T_{\bullet,\bullet}}{S/b}$$
 =  $HA \tan \beta$ 

The actual basic, additional, and total lifts for any section of the wing may then be obtained from

$$l_b = c_{l_b} qc$$

$$l_a = C_L c_{l_{a1}} qc$$

$$l = c_l qc$$

Values of l may be computed for the various spanwise stations and the curve of the span lift-distribution may be plotted. Typical semispan lift-distribution curves are shown in figure 10.

The semispan induced angle-of-attack distribution may be obtained from

$$\alpha_{i_a} = \alpha_a - \frac{c_l}{a_0}$$

where

$$\alpha_a = \alpha_{a_s} + \frac{y}{b/2}\epsilon$$

$$\alpha_{a_s} = \frac{C_L}{a} + J\epsilon$$

The remaining characteristics are obtained simply by finding the required factor for the desired values of  $c_l/c_s$  and A from the charts and by computing the characteristics from the formulas previously given, using the average value of  $a_0$  where  $a_0$  is required. The formulas are summarized here for convenience.

Lift-curve slope:

$$a = f \frac{a_0}{1 + \frac{57.3a_0}{\pi A}}$$

Angle of attack corresponding to any  $C_{\rm L}$ :

$$\alpha_{s} = \frac{C_{L}}{a} + \alpha_{l_{0}} + J\epsilon$$

Angle of zero lift:

$$\alpha_{l_{(L=0)}} = \alpha_{l_{0}} + J\epsilon$$

Induced-drag coefficient:

$$C_{D_i} = \frac{C_L^2}{\pi A u} + C_L \epsilon a_0 v + (\epsilon a_0)^2 w$$

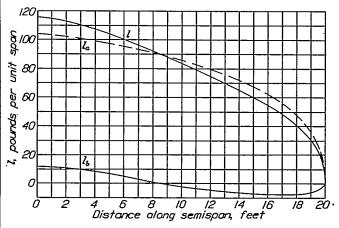


Figure 10.—Typical semispan lift distribution.  $C_L=1.2$ .

Pitching-moment coefficient about an axis through the aerodynamic center:

$$\begin{split} C_{m_a.c.} &= C_{m_b} + C_{m_{l_b}} \\ C_{m_b} &= EC_{m_a.c.} \\ C_{m_{l_b}} &= -G\epsilon a_0 A \tan \beta \end{split}$$

Aerodynamic-center position (x coordinate):

$$\frac{x_{a.c.}}{S/b} = HA \tan \beta$$

Although  $C_{m_{\theta}}$  may usually be determined from the foregoing formula, equation (4) should be used if  $c_{m_{\theta},c}$  varies considerably across the span.

Illustrative example.—In order to illustrate the method of using the charts, an example will be worked

out for a wing with straight taper and rounded tips having the following characteristics:

$$A=6$$
 $c_1/c_2=0.5$ 
 $b=40$  feet
 $S=266.7$  sq. ft.
 $\beta=10^{\circ}$ 
 $C_L=1.2$ 
 $q=10$  lb./sq. ft.

Construction tip section: Root section: N. A. C. A. 2409 N. A. C. A. 4415  $\begin{array}{c} a_{0t} = 0.099 \\ \alpha_{l_0t} = -1.7^{\circ} \\ c_{m_a.c._t} = -0.044 \end{array}$  $a_{0_s} = 0.097$   $\alpha_{l_{0_s}} = -3.8^{\circ}$   $c_{m_{a,c,s}} = -0.083$ 

The angle of twist measured between the chords of the root and construction tip sections is  $-5^{\circ}$  (washout). Then, by the use of the angles of zero lift of the root and tip sections and by reference to figure 1, the angle of aerodynamic twist is determined to be  $-7.1^{\circ}$ .

The chord at several stations along the semispan and the calculation of the lift distribution are given in table IV. In the table,  $a_0$  and  $c_{m_{a,c}}$  are assumed to have a linear variation along the semispan. Values of  $L_b$  and  $L_a$  were obtained from tables I and II for an aspect ratio of 6 and a taper ratio of 0.5 and the basic, additional, and total lift distributions were computed and plotted in figure 10. The pitching-moment coefficient  $c_{m_{a.e.}}$  varies so much along the semispan that  $C_{m_s}$  cannot be found by use of the factor E but must be found from (4). Accordingly,  $c_{m_{a,c}}c^2$  is plotted against y in figure 11 and  $C_{m_{\theta}}$  is found from the area under the curve to be -0.072.

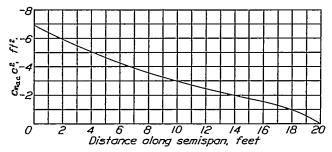


FIGURE 11.—Graphical determination of section pitching moment.  $C_{m_0} = \frac{2b}{S^2} \int_0^{b/2} c_{m_0,p} c^2 dy = -0.072$ 

From figures 2 to 9 and the equations on page 7 the remaining factors and characteristics are determined to be

Method for wing of special form.-If it is desired to find the characteristics of a wing having a chord distribution that lies between the chord distributions of the tapered and elliptical wings, such as a wing with a constant-chord center section, an interpolation may be made between the values for the tapered and elliptical wings to find most of the characteristics.

The lift distribution for such wings may be found by an approximate method that has been tried for a few wings having parallel center sections and has given satisfactory results. The method has been taken from reference 5 with the symbols converted to the notation of this report. Approximate values of  $L_a$ , which will be designated  $L_{a'}$ , may be calculated from

$$L_{a'} = rac{\sqrt{1 - \left(rac{y}{b/2}
ight)^2}}{\sqrt{1 - \left(rac{y}{b/2}
ight)^2}} + rac{3}{8}} \left(rac{A}{2}lpha_a + rac{1}{\pi}
ight)$$

where

$$\alpha_a = \frac{8}{\pi A} \left[ \left( \frac{\sqrt{1 - \left(\frac{y}{b/2}\right)^2}}{\frac{m_0 c}{b/2}} \right)_{\text{mean}} + \frac{1}{8} \right]$$

The procedure is to choose a number of points at convenient intervals along the semispan (12 points should be sufficient for the usual plan forms); then from the

values of 
$$c$$
 at those points the mean value of  $\frac{\sqrt{1-\left(\frac{y}{b/2}\right)^2}}{\frac{m_0c}{b/2}}$ 

is calculated. The value of  $\alpha_a$  may then be found and from the values of y and c,  $L_{a}'$  at each point along the semispan may be computed. The values of  $L_a'$  should correspond to a  $C_L$  approximately equal to 1. The actual  $C_L$  may be found from

$$C_{L} = \int_{0}^{1} L_{a}' d\left(\frac{y}{b/2}\right)$$

 $C_L {=} \int_0^1 L_a' d \Big( \frac{y}{b/2} \Big)$  and  $C_L$  may be conveniently found from the area under a curve of  $L_{a'}$  plotted against  $\frac{y}{b/2}$ . Finally,  $L_{a}$  may be found from  $L_a=L_a'/C_L$ . Values of  $c_{la1}$  may then be calculated by the previously indicated method and, if desired,  $C_{D_t}$  and  $\frac{x_{a.\ c.}}{S/b}$  may be found from equations (1) and (2).

If a wing has considerable dihedral or a curved wing axis, an integration may be made directly from the section characteristics. For this purpose, the best procedure would be to resolve the section values  $c_{i_0}$ and  $c_{d_0}$  into components along and parallel to the xand z axes, where the z axis is perpendicular to the x axis and lies in the plane of symmetry. Owing to dihedral, there will be a vertical coordinate of the aero-

dynamic center and a pitching moment about the aerodynamic center of the force components in the x direction. The coordinates of the aerodynamic center and of the pitching moment about it may be found from integrations like (2) and (3) by substituting the appropriate values of the x and z force components. For example,  $x_{a,c}$  would be found from

$$x_{a.c.} = \frac{\frac{2}{S} \int_0^{b/2} c_{z_a} cxdy}{C_{z_a}}$$

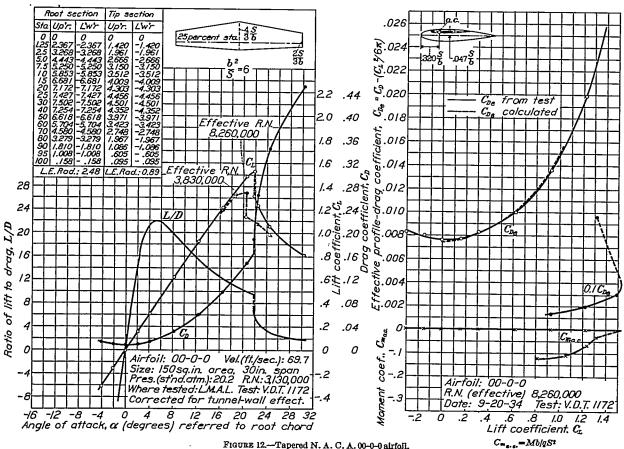
where

$$C_{z_a} = \frac{2}{S} \int_0^{b/2} c_{z_a} c dy$$

to the desired angle of twist and the sections between the root and tip were then formed by using straight lines between corresponding stations of the root and tip sections. Formation of the wings in this manner results in a nonlinear distribution of twist along the semispan. In plan view the quarter-chord points of the sections lie on a straight line across the semispan; the sweepback was measured between this line and the lateral axis.

Three different amounts of sweepback, 0°, 15°, and 30°, and three types of airfoil sections, symmetrical, cambered, and reflexed, were used.

As the wings differ primarily in airfoil section, The values of  $x_{a.e.}$  and  $C_{z_a}$  may be found by plotting | sweepback, and twist, a convenient designating number



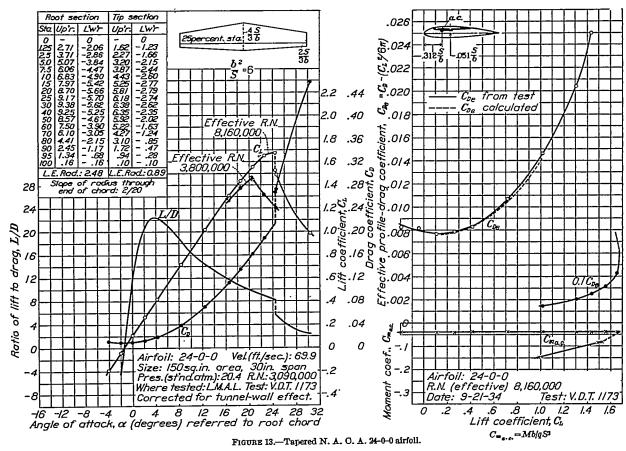
 $c_{z_a}c_x$  and  $c_{z_a}c$  against the distance along the semispan | was used to distinguish the wings, such as 24-30-8.50. and finding the area under the curves.

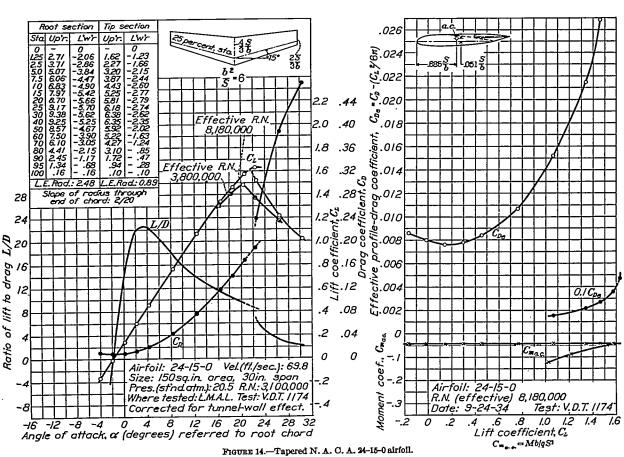
## TESTS OF TAPERED WINGS

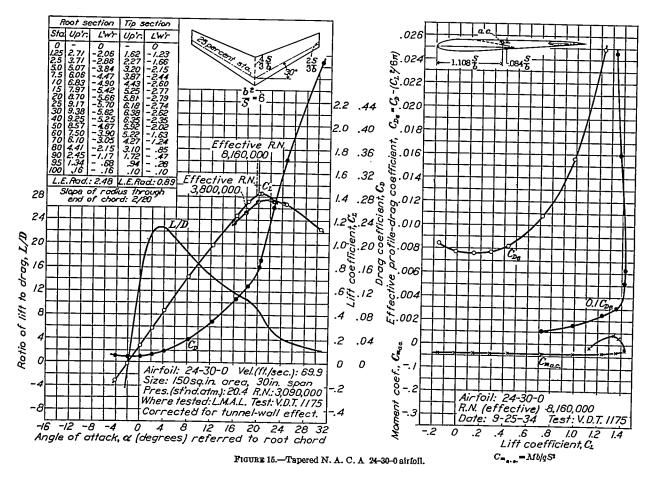
In order to provide test data on tapered wings, including wings with sweepback and twist, and to provide a check on the previously outlined method of computing characteristics, nine tapered wings were tested. The plan forms and sections of the wings are shown in figures 12 to 20. The aspect ratio of all the wings was 6; the taper ratio of eight of the wings was 0.5 and of one wing was 0.25. For all the wings the thickness ratio of the root section was 15 percent and

In this number 24 designates the N. A. C. A. airfoil mean line, i. e., 2 means 0.2 chord maximum camber and 4 that the maximum camber is at 0.4 chord; 30 gives the sweepback in degrees; and 8.50 gives the washout in degrees.

The wings are listed in table V. The first two wings have no sweepback and no twist and differ only in airfoil section. The next two have increased sweepback. The five remaining wings are examples of various methods of combining sweepback, twist, and airfoil section to obtain wings having a small positive pitching moment; such wings would be suitable for of the tip sections 9 percent. The tip section was set | tailless airplanes. The amounts of twist and of 512







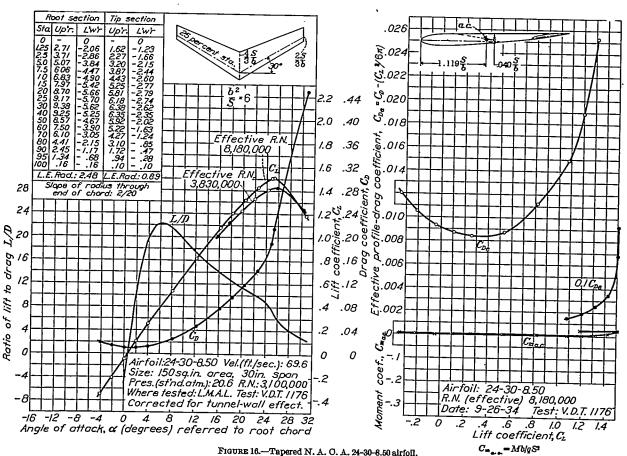
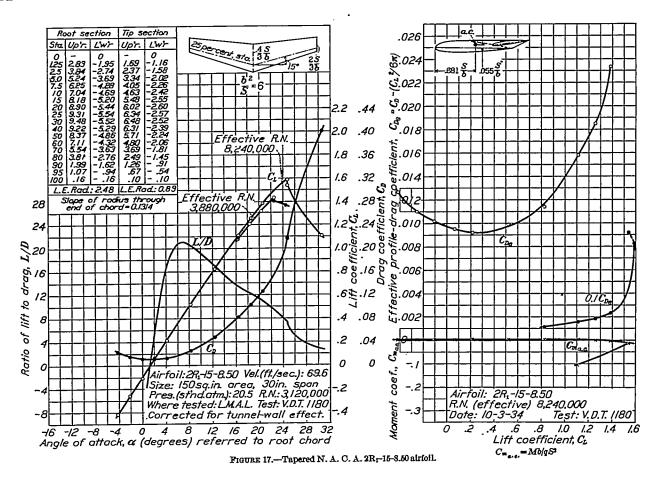


FIGURE 16.-Tapered N. A. O. A. 24-30-8.50 airfoil.



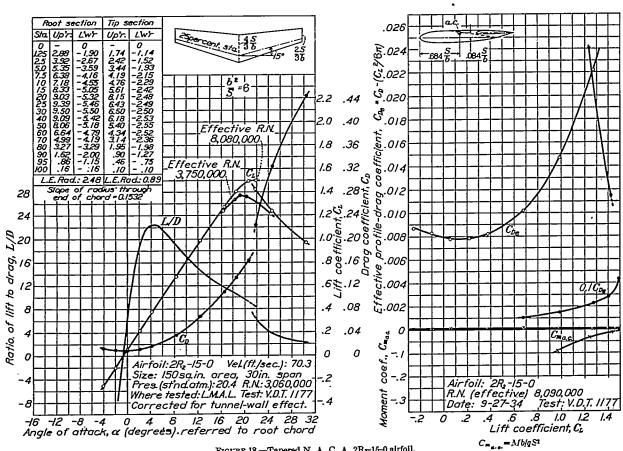
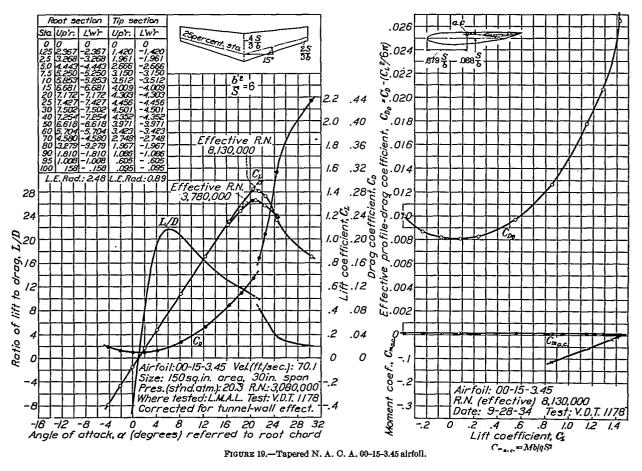


FIGURE 18.-Tapered N. A. C. A. 2Rz-15-0 airfoil.



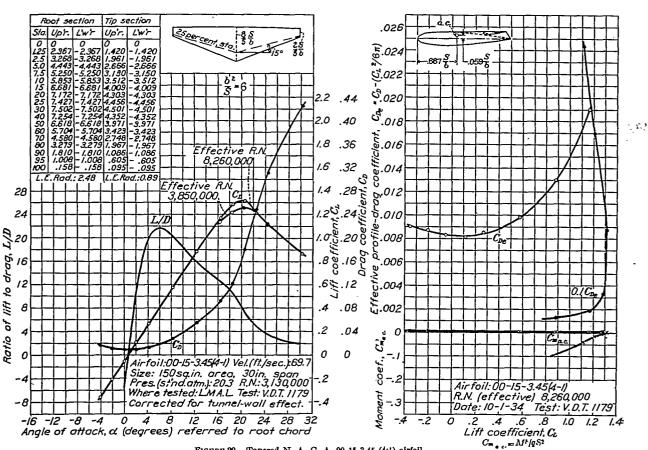


FIGURE 20.-Tapered N. A. O. A. 00-15-3.45 (4:1) airfoil.

sweepback necessary to obtain the desired pitching moment were determined by the method previously given for computing pitching moments, except that data for wings with trapezoidal tips were used. The 24-30-8.50 wing has sufficient twist to obtain the desired pitching moment using a cambered section and 30° sweepback. The  $2R_1$ -15-8.50 wing has the same twist but half the sweepback and a reflexed airfoil section to obtain a positive pitching moment. The  $2R_2$ -15-0 airfoil has no twist and increased reflex. A symmetrical section together with twist is used for the 00-15-3.45 wing, while the last wing has the same twist and sweepback as the previous wing but a taper ratio of 0.25.

The variable-density wind tunnel in which the tests were made is described in reference 6 together with the method of making tests. The lift, drag, and pitching moment of the wings were measured at a tank pressure of 20 atmospheres.

The results of the tests, corrected for tunnel-wall effect, are given in the form of dimensionless coefficients and are plotted in figures 12 to 20. The lift-curve peak is given for two values of effective Reynolds Number to indicate the scale effect. The effective Reynolds Number, at which the maximum lift coefficients apply in flight, is the test Reynolds Number multiplied by a turbulence factor, 2.64.

In order to make possible a more accurate reading of drag coefficients than can be made from the plots against angle of attack, a drag coefficient has been plotted against lift coefficient with the induced drag for elliptical span loading deducted; that is

$$C_{D_{\epsilon}} = C_{D} - \frac{C_{L}^{2}}{\pi A}$$

The coefficient  $C_{D_e}$  is called the "effective profile-drag coefficient" and is useful for comparing the drag of tapered wings, as it includes with the true profile drag any additional induced drag caused by a departure from the ideal elliptical lift distribution. Notice should be taken that  $C_{D_e}$  cannot be used like a profile-drag coefficient to compute the effect of change of aspect ratio but applies only to the particular wings tested. The values of  $C_{D_e}$  have been corrected to the effective Reynolds Number (references 7 and 8) by allowing for the reduction in skin-friction drag due to the change from the test to the effective Reynolds Number. The reduction amounted to  $C_{D}=0.0011$ .

The pitching-moment coefficients plotted against the lift coefficient are given about an axis through the aerodynamic center of the wings in order to obtain a practically constant value of pitching-moment coefficient. The aerodynamic center was determined from the slope of the test pitching-moment curve. The location of the aerodynamic center is given on the plots by its distance from the leading edge and above the chord of the root section. These distances are given as fractions of the ratio of area to span, S/b.

The shapes of the lift and pitching-moment curves near maximum lift provide information on the nature of the stalling of the wings. The 24-0-0 wing has a sharp drop in lift after the maximum, indicating that stalling occurs almost simultaneously over a considerable portion of the wing. Also the  $C_{m_a,c}$ after the stall is like that of normal wings. In contrast to this wing, the 24-30-0 wing, which has the same airfoil sections but 30° sweepback, has a rounded lift-curve peak, indicating that stalling occurs progressively along the span. The pitching-moment coefficient is positive after the stall, which shows that stalling begins at sections behind the aerodynamic center. Washout, as in the case of the 24-30-8.50 wing, reduces the tendency to stall of sections behind the aerodynamic center, which may be verified by reference to the  $C_{m_{a,c}}$  curve. Stalling, however, still begins behind the aerodynamic center, as the  $C_{m_{a,a}}$  is positive after the stall. All the wings, except the 24-30-0 and 24-30-8.50, are stable after the stall.

The important test results for all the wings are summarized in table V. The coordinates of the aerodynamic center are expressed as fractions of S/b. The 24-0-0, 24-15-0, and 24-30-0 wings show a decrease of  $C_{L_{max}}$  as the sweepback is increased. For the 24-30-8.50 wing, the effect of sweepback is partly compensated by twist, which reduces the tendency to stall of the low Reynolds Number sections near the tips and therefore increases  $C_{L_{max}}$ . The drag, however, is also increased. Of the wings designed to have a small positive  $C_{m_0}$ , the  $2R_2$ -15-0 wing has the highest ratio of  $C_{L_{max}}/C_{D_{\sigma_{min}}}$ .

## COMPARISON OF TEST AND CALCULATED RESULTS

Pitching-moment characteristics, lift-curve slope, and drag.—The lift distribution and other theoretical data used to determine the desired pitching-moment coefficient of the wings are now used to predict other characteristics. In addition to  $C_{m_0}$ , the aerodynamiccenter position, the angle of zero lift, and the lift-curve slope have been calculated. The values of a were calculated from the formula in figure 2. In this formula a value of  $a_0$  corresponding to the  $a_0$  for the N. A. C. A. 0012 and 2412 sections at a Reynolds Number of 3,000,000 was used, inasmuch as the effect of variations of  $a_0$  with section and Reynolds Number is small. As the value of  $a_0$  used in the formula was derived from tests of rectangular wings, a correction for square tips has been applied in order to obtain a better value of the section lift-curve slope. The correction, derived from tests of wings with rounded tips, is given in reference 9.

The calculated values of the pitching-moment coefficient at zero lift, the aerodynamic-center position, the angle of zero lift, and the lift-curve slope are generally in good agreement with the test values (table VI). The agreement of the pitching-moment coefficient at zero lift and the aerodynamic-center position, which are

calculated from the basic and additional lift distributions, respectively, indicate that the theoretical lift distributions must also agree reasonably well with the actual distributions.

In addition to the foregoing characteristics, the drag has been calculated for the 00-0-0 and 24-0-0 airfoils. The comparison between calculation and experiment is based on values of the effective profile-drag coefficient. The calculated values were obtained from

$$C_{D_{\delta}} = \frac{2}{S} \int_{0}^{b/2} c_{d_0} c dy + C_{D_{\delta}} - \frac{C_L^2}{\pi A}$$

In order to find the value of the integral, values of  $c_{d_0}$  were determined as follows at several points along the semispan for convenient values of total wing  $C_L$ . For each value of  $C_L$  the distribution across the semispan of  $c_l$ , Reynolds Number, and thickness ratio were calculated. Then, for each point on the semispan,  $c_{d_0}$  was found for the appropriate  $c_l$ , Reynolds Number,

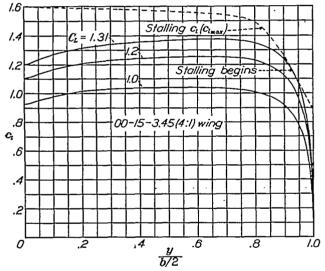


FIGURE 21.—Determination of the CL at which a tapered wing begins to stall.

and thickness ratio, using data that are expected to be published soon in a report concerning scale effect on airfoils. From the values of  $c_{d_0}$ , a curve of  $c_{d_0}c$  was plotted against y and the value of the integral was determined from the area under the curve. The value of  $C_{D_t}$  was obtained for the formula previously given. The calculated and test values of  $C_{D_s}$  are compared in figures 12 and 13. The agreement is considered good.

Estimation of maximum lift coefficient.—A final characteristic to be estimated is the maximum lift coefficient, which should be nearly equal to the  $C_L$  at which stalling begins. The method of determining the  $C_L$  at which stalling begins is demonstrated for the 00–15–3.45 (4:1 taper) wing in figure 21. The lift coefficient at which each section along the semispan stalls (shown by the dashed curve) was obtained by

using the maximum lift coefficients of the symmetrical sections given in reference 10 but with the values of  $C_{L_{max}}$  increased 3 percent. This correction was made for the same reason that  $a_0$  was corrected; that is, to allow for the effect of square tips and thereby to obtain a closer approach to true section characteristics. Better section characteristics will be obtained as a result of an investigation in progress but the correction used is sufficiently accurate for the present purpose. As the values of  $C_{L_{max}}$  given in reference 10 were for a Reynolds Number of 3,000,000, correction increments were applied to correct the values of  $C_{L_{max}}$  to the actual Reynolds Number of each section along the span. Correction increments applying to various airfoil sections are expected to be published in the previously mentioned report concerning scale effect on airfoils.

The curves of  $c_i$  distribution for several values of wing  $C_L$  given in figure 21 were determined by the method previously given for finding  $c_i$  distribution. As soon as the  $c_i$  curve becomes tangent to the stalling  $c_{l_{max}}$  curve, the section at that point reaches its maximum lift coefficient and stalling should soon spread over a considerable part of the wing. Thus, for the 00-15-3.45 (4:1 taper) wing, stalling is indicated as beginning near the tips, at a  $C_L$  of 1.31. Stalling, however, is so close to the tip that it may be modified by the tip vortex. The measured  $C_{L_{max}}$  is 1.32, but this value is probably low owing to the sweepback of the wing. This method, when applied to several other tapered wings without sweepback but having various taper ratios and aspect ratios, gave a stalling  $C_{L}$  that was within a few percent of the measured  $C_{L_{max}}$  for all the wings; therefore, the method should prove useful for estimating the  $C_{L_{max}}$  of tapered wings.

The 00-15-3.45 (4:1 taper) wing is an example of the harmful effect of excessive taper on  $C_{L_{max}}$ . Large taper not only tends to cause a low  $C_{L_{max}}$  but also tends to cause stalling near the tips, which results in poor lateral control at low speeds. Improvement could be obtained by using less taper and thicker sections near the tips.

Although all of the characteristics of tapered wings have not yet been satisfactorily calculated, it may be concluded that the following important aerodynamic characteristics—angle of zero lift, the lift-curve slope, the pitching-moment coefficient, the aerodynamic-center position, and the span lift distribution—can be calculated with sufficient accuracy for engineering purposes.

Langley Memorial Aeronautical Laboratory, National Advisory Committee for Aeronautics, Langley Field, Va., May 1, 1936.

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TABLE I.—BASIC SPAN LIFT-DISTRIBUTION DATA VALUES OF  $L_b$  FOR TAPERED WINGS WITH ROUNDED TIPS  $c_{lb} = \frac{e a_0 S}{c b} L_b$ 

	cb													
A cylc.	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0			
		SPANWISE STATION $\frac{y}{b/2}=0$												
2	-0. 118 153 183 211 235 256 274 304 329 350 367 384 398	-0. 121 160 192 221 248 269 288 318 342 364 380 399 411	-0. 122 - 162 - 197 - 224 - 233 - 275 - 283 - 322 - 350 - 370 - 386 - 405 - 417	-0. 122 163 199 225 263 276 293 323 349 370 385 403 415	-0. 122 165 199 225 252 274 291 348 388 382 400 410	-0. 121 164 199 225 252 272 280 320 345 379 393 404	-0. 121 164 198 224 250 270 288 318 341 375 387 389	-0. 121 163 197 224 247 263 285 315 337 355 370 380 392	-0. 120 162 196 221 244 254 331 331 350 362 386	-0. 120 161 194 219 243 261 279 305 323 342 358 368 378	-0. 120 160 192 218 242 258 276 299 317 334 360			
		SPANWISE STATION $\frac{y}{b/2}$ =0.2												
2	-0.076098117131145168182197206212219222	-0.080108108148162178189207276242242247255	-0.082111135156173189200239239248256260269	-0.085112138169176192204240240258258271	-0.086113137150176192204225239248257235271	-0.086113137188176192205225238248256271	-0.086113137168176191205226238248256272	-0. 085 113 137 188 176 191 206 226 238 248 256 272	-0.085 - 112 137 137 175 190 205 225 237 248 256 265 272	-0. 084 110 136 156 172 190 204 225 237 248 256 272	0.083108132152189204225237248262370			
				SPA	nwise st	ATION b/	2-0.4			,				
2	-0.006 002 0 .004 .009 .012 .014 .021 .028 .038 .043 .049	-0,011 -,010 -,006 -,004 -,002 -,001 0 .007 .009 .013 .019 .022 .023	-0. 013 012 011 010 008 010 008 002 001 0 . 002 . 004 . 006	-0. 015 015 015 012 012 013 012 010 010 010 008 008 008	-0. 016 016 016 016 017 017 017 017 018 017 018 017	-0. 016 016 016 018 018 018 018 019 020 021 022 022 022	-0. 016 018 018 018 020 020 021 022 025 025 025 025 031	-0. 016 016 019 021 021 022 022 027 029 031 034 038	-0. 016 017 020 021 022 025 029 030 032 038 041	-0. 016 018 020 022 024 027 030 032 038 040 041 043 046	0.015018021023029030032032045046			

TABLE I.—BASIC SPAN LIFT-DISTRIBUTION DATA—Continued VALUES OF  $L_b$  FOR TAPERED WINGS WITH ROUNDED TIPS  $c_{l_b} = \frac{\epsilon a_0 S}{cb} L_b$ 

cu/c.	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
				SPA	nwise st	CATION b/							
2	0. 052 . 070 . 085 . 099 . 109 . 119 . 128 . 139 . 148 . 165 . 160 . 165 . 170	0. 052 . 069 . 082 . 095 . 107 . 117 . 122 . 128 . 145 . 162 . 158 . 162 . 169	0. 051 . 068 . 081 . 092 . 104 . 114 . 121 . 135 . 141 . 150 . 154 . 160	0. 050 . 088 . 080 . 091 . 102 . 112 . 120 . 132 . 140 . 148 . 151 . 168	0. 050 . 088 . 080 . 091 . 101 . 111 . 120 . 131 . 140 . 145 . 149 . 152	0. 050 . 068 . 080 . 091 . 101 . 110 . 130 . 139 . 142 . 146 . 148	0.050 .068 .060 .091 .100 .110 .119 .120 .137 .141 .143 .145	0. 050 . 068 . 080 . 091 . 100 . 110 . 118 . 129 . 135 . 140 . 141 . 142 . 143	0. 049 . 068 . 080 . 090 . 100 . 110 . 118 . 128 . 134 . 139 . 140 . 140	0. 049 . 068 . 080 . 090 . 100 . 109 . 117 . 126 . 132 . 138 . 139 . 140	0. 048 .068 .080 .090 .100 .108 .116 .124 .130 .135 .136 .138		
	. SPANWISE STATION $\frac{y}{b/2}$ =0.8												
2	0. 072 . 088 . 100 . 109 . 115 . 121 . 126 . 136 . 145 . 152 . 159 . 161 . 168	0. 079 . 088 . 113 . 125 . 135 . 142 . 149 . 160 . 170 . 182 . 186 . 197 . 201	0. 080 . 101 . 120 . 135 . 148 . 168 . 164 . 178 . 188 . 200 . 205 . 215 . 220	0. 082 . 102 . 123 . 138 . 162 . 163 . 174 . 188 . 200 . 210 . 224 . 232	0. 083 . 104 . 125 . 140 . 156 . 169 . 180 . 195 . 208 . 218 . 222 . 230 . 237	0. 085 . 108 . 123 . 143 . 160 . 172 . 182 . 200 . 212 . 221 . 229 . 235 . 241	0. 085 . 109 . 128 . 147 . 160 . 173 . 182 . 201 . 214 . 223 . 232 . 239 . 245	0. 086 - 110 - 130 - 148 - 162 - 173 - 183 - 202 - 216 - 227 - 233 - 242 - 248	0. 086 . 110 . 130 . 148 . 163 . 174 . 183 . 203 . 216 . 228 . 228 . 243 . 248	0. 084 . 108 . 130 . 148 . 164 . 174 . 184 . 201 . 214 . 225 . 232 . 242 . 248	0, 081 106 129 149 165 175 184 198 210 220 229 238 247		
	SPANWISE STATION $\frac{y}{b/2}$ =0.9												
2	0. 059 . 068 . 074 . 081 . 087 . 090 . 092 . 098 . 100 . 102 . 103 . 105 . 107	0. 068 . 083 . 098 . 107 . 117 . 123 . 131 . 139 . 147 . 156 . 161 . 166 . 172	0. 072 . 092 . 111 . 122 . 136 . 146 . 153 . 166 . 178 . 188 . 197 . 202 . 211	0. 073 . 068 . 118 . 131 . 148 . 160 . 170 . 184 . 198 . 208 . 219 . 228 . 233	0. 075 . 099 . 121 . 138 . 154 . 167 . 179 . 197 . 210 . 220 . 231 . 243 . 248	0.076 .100 .122 .140 .159 .171 .182 .201 .218 .231 .241 .252 .260	0. 075 . 100 . 123 . 141 . 160 . 171 . 183 . 203 . 221 . 238 . 249 . 260 . 268	0. 075 - 100 - 123 - 141 - 160 - 172 - 184 - 225 - 225 - 241 - 253 - 263 - 273	0. 075 - 100 - 123 - 142 - 160 - 172 - 185 - 207 - 228 - 243 - 258 - 269 - 279	0. 075 - 100 - 123 - 142 - 160 - 172 - 186 - 229 - 229 - 245 - 259 - 271 - 282	0, 075 . 100 . 123 . 142 . 160 . 172 . 187 . 210 . 230 . 246 . 280 . 275 . 285		
	<u> </u>			SPA	nwise st	ATION b	2-0.95						
2	0. 038 . 044 . 050 . 052 . 054 . 056 . 057 . 088 . 059 . 060 . 061 . 061	0. 051 . 053 . 072 . 083 . 088 . 093 . 100 . 107 . 112 . 116 . 121 . 126 . 128	0. 058 . 073 . 076 . 100 . 109 . 116 . 125 . 138 . 143 . 151 . 159 . 166 . 173	0. 059 . 078 . 092 . 107 . 119 . 130 . 140 . 162 . 165 . 174 . 184 . 194 . 203	0. 060 . 079 . 095 . 110 . 122 . 135 . 146 . 162 . 179 . 190 . 203 . 213 . 225	0. 080 . 080 . 097 . 112 . 123 . 140 . 152 . 171 . 189 . 202 . 218 . 229 . 239	0. 060 . 080 . 099 . 113 . 120 . 144 . 158 . 178 . 198 . 211 . 222 . 236 . 245	0.060 .080 .100 .114 .132 .148 .160 .182 .200 .215 .229 .241	0. 059 . 080 . 100 . 116 . 132 . 150 . 181 . 188 . 202 . 218 . 233 . 248 . 259	0, 059 . 079 . 100 . 117 . 131 . 149 . 160 . 187 . 205 . 221 . 236 . 251 . 265	0. 058 .078 .059 .116 .130 .145 .159 .183 .204 .222 .238 .255 .271		
				SPA	NWISE 87	FATION b	2 0.975						
2	0. 019 . 022 . 026 . 029 . 030 . 030 . 031 . 031 . 031 . 031 . 032 . 032	0. 030 . 039 . 043 . 051 . 055 . 060 . 062 . 067 . 069 . 077 . 077 . 083 . 086	0. 035 .045 .054 .065 .071 .078 .081 .090 .095 .102 .111 .121	0. 037 .049 .060 .070 .079 .087 .091 .105 .115 .127 .138 .150	0. 037 .050 .062 .071 .082 .091 .100 .115 .131 .143 .155 .169	0.037 .051 .064 .075 .088 .098 .107 .124 .141 .155 .169 .182	0.037 .052 .068 .078 .091 .101 .112 .132 .149 .163 .178 .191	0. 036 .054 .069 .081 .094 .107 .120 .138 .153 .171 .182 .197 .208	0. 038 .053 .069 .082 .097 .110 .121 .141 .180 .175 .188 .200 .210	0. 035 .052 .068 .088 .087 .110 .121 .142 .161 .177 .190 .201	0. 034 .051 .067 .083 .097 .110 .121 .143 .162 .178 .191 .202 .213		

REPORT NO. 572 NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TABLE II.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA VALUES OF  $L_a$  FOR TAPERED WINGS WITH ROUNDED TIPS,  $c_{l_{a1}} = \frac{S}{cb} L_a$ 

c.jc.													
A	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0		
				SI	ANWIS	E STATI	ON # -	0			-		
2	1. 439 1. 489 1. 527 1. 559 1. 585 1. 609 1. 629 1. 661 1. 708 1. 726 1. 741 1. 755	1. 400 1. 430 1. 452 1. 473 1. 492 1. 510 1. 534 1. 558 1. 578 1. 592 1. 610 1. 623 1. 632	1. 367 1. 385 1. 400 1. 414 1. 428 1. 440 1. 456 1. 473 1. 490 1. 502 1. 513 1. 525 1. 531	1. 339 1. 350 1. 369 1. 369 1. 378 1. 392 1. 409 1. 429 1. 429 1. 433 1. 441 1. 446	1. 316 L 322 1. 329 L 333 L 338 L 340 L 344 L 355 L 361 L 368 L 370 L 372	1. 301 1. 302 1. 302 1. 301 1. 300 1. 300 1. 300 1. 306 1. 308 1. 309 1. 309 1. 309 1. 308	1. 298 1. 279 1. 277 1. 272 1. 267 1. 264 1. 264 1. 261 1. 260 1. 255 1. 252 1. 250	1. 292 1. 275 1. 260 1. 248 1. 237 1. 232 1. 229 1. 222 1. 214 1. 203 1. 203 1. 199	1. 290 1. 203 1. 242 1. 225 1. 211 1. 203 1. 198 1. 187 1. 180 1. 172 1. 165 1. 160 1. 152	1. 287 1. 253 1. 226 1. 204 1. 187 1. 176 1. 165 1. 152 1. 138 1. 138 1. 127 1. 118 1. 109	1. 282 1. 246 1. 211 1. 186 1. 163 1. 149 1. 135 1. 120 1. 109 1. 100 1. 090 1. 080 1. 070		
	SPANWISE STATION $\frac{y}{b/2}$ =0.2												
2	L 389 1. 405 L 434 L 477 L 491 1. 502 L 513 L 520 L 527 L 532 L 539 L 547	1. 329 1. 346 1. 363 1. 377 1. 393 1. 401 1. 411 1. 417 1. 423 1. 428 1. 429 1. 431	1. 300 1. 308 1. 318 1. 324 1. 329 1. 332 1. 338 1. 347 1. 354 1. 354 1. 358 1. 359 1. 360	1. 279 1. 279 1. 284 1. 288 1. 290 1. 291 1. 294 1. 209 1. 307 1. 303 1. 309 1. 311	1. 267 1. 260 1. 260 1. 260 1. 260 1. 259 1. 259 1. 261 1. 265 1. 268 1. 269 1. 270 1. 271	1. 260 1. 248 1. 243 1. 240 1. 236 1. 236 1. 236 1. 236 1. 232 1. 232 1. 232 1. 231 1. 230	1. 258 1. 241 1. 232 1. 223 1. 218 1. 214 1. 212 1. 209 1. 202 1. 201 1. 199 1. 195 1. 190	1. 256 1. 234 1. 220 1. 208 1. 200 1. 193 1. 189 1. 182 1. 170 1. 164 1. 160 1. 165	1. 253 1. 228 1. 209 1. 104 1. 184 1. 174 1. 168 1. 168 1. 148 1. 144 1. 135 1. 130 1. 123	1. 250 1. 221 1. 198 1. 181 1. 169 1. 157 1. 148 1. 137 1. 128 1. 129 1. 110 1. 103 1. 098	1. 248 1. 214 1. 186 1. 163 1. 151 1. 138 1. 129 1. 114 1. 102 1. 094 1. 097 1. 078 1. 069		
		SPANWISE STATION $\frac{y}{b/2}$ =0.4											
2	1. 217 1. 220 1. 223 1. 229 1. 229 1. 229 1. 228 1. 228 1. 228 1. 228 1. 228 1. 228 1. 228 1. 228	1. 190 1. 191 1. 192 1. 193 1. 193 1. 192 1. 192 1. 192 1. 191 1. 189 1. 186 1. 182	1. 178 1. 176 1. 173 1. 173 1. 171 1. 170 1. 168 1. 167 1. 166 1. 161 1. 158 1. 152 1. 149	L. 172 1. 168 1. 162 1. 159 1. 155 1. 155 1. 150 1. 148 1. 145 1. 136 1. 131 1. 129 1. 127	L 172 L 161 L 156 L 149 L 145 L 138 L 132 L 132 L 112 L 111 L 110	1. 171 1. 160 1. 151 1. 142 1. 138 1. 131 1. 128 1. 121 1. 111 1. 104 1. 101 1. 100 1. 098	L 170 L 159 1. 149 1. 140 1. 132 1. 124 1. 120 1. 113 1. 107 1. 100 1. 097 1. 092 1. 089	1. 169 1. 158 1. 148 1. 138 1. 128 1. 121 1. 116 1. 108 1. 102 1. 096 1. 091 1. 083	1. 169 1. 157 1. 147 1. 136 1. 127 1. 120 1. 113 1. 104 1. 099 1. 090 1. 086 1. 080 1. 078	1. 168 1. 156 1. 146 1. 134 1. 126 1. 119 1. 111 1. 102 1. 087 1. 081 1. 076	1. 168 1. 185 1. 145 1. 133 1. 125 1. 118 1. 110 1. 100 1. 090 1. 082 1. 075 1. 070 1. 085		
		-		sı	RIWNA	E STATI	ON $\frac{y}{b/2}$	0. 6			_		
2	0. 970 . 950 . 932 . 920 . 909 . 891 . 881 . 868 . 868 . 851	0. 976 .962 .948 .938 .930 .920 .916 .907 .901 .895 .888 .833 .876	0. 984 - 975 - 963 - 953 - 949 - 940 - 938 - 929 - 923 - 918 - 912 - 906 - 898	0. 992 - 985 - 978 - 971 - 966 - 959 - 958 - 947 - 941 - 937 - 931 - 925 - 920	1. 003 . 996 . 992 . 988 . 981 . 975 . 972 . 961 . 963 . 953 . 948 . 944 . 940	1. 010 1. 004 1. 002 1. 000 .993 .989 .976 .972 .969 .963 .963	1. 012 1. 011 1. 008 1. 008 1. 002 1. 000 999 992 989 986 983 981 978	1, 014 1, 018 1, 014 1, 015 1, 013 1, 012 1, 011 1, 003 1, 003 1, 000 998 995	1, 016 1, 023 1, 023 1, 024 1, 024 1, 024 1, 024 1, 023 1, 022 1, 019 1, 017 1, 015 1, 012	1, 018 1, 030 1, 035 1, 038 1, 039 1, 039 1, 039 1, 039 1, 035 1, 035 1, 035 1, 032 1, 032	1, 019 1, 038 1, 050 1, 053 1, 055 1, 054 1, 053 1, 052 1, 045 1, 049 1, 048 1, 047 1, 046		
				81	ANWIS	E STATI	ON #-	0.8					
2	0. 615 . 559 . 568 . 548 . 531 . 517 . 504 . 486 . 472 . 462 . 456 . 450 . 444	0. 678 - 659 - 644 - 632 - 619 - 609 - 500 - 576 - 576 - 589 - 559 - 545	0. 712 . 700 . 691 . 635 . 675 . 670 . 663 . 653 . 648 . 641 . 638 . 636	0. 731 - 726 - 723 - 720 - 717 - 713 - 710 - 704 - 702 - 699 - 698 - 698 - 698 - 698	0. 740 . 743 . 748 . 750 . 753	0. 745 . 754 . 764 . 769 . 775 . 778 . 779 . 783 . 788 . 789 . 791 . 796 . 801	0. 746 . 764 . 781 . 790 . 800 . 802 . 808 . 815 . 821 . 825 . 830 . 835 . 842	0. 748 .772 .795 .808 .820 .827 .834 .842 .860 .863 .863 .870 .878	0. 747 . 782 . 806 . 822 . 838 . 845 . 854 . 868 . 877 . 887 . 894 . 901 . 909	0. 747 . 790 . 816 . 834 . 851 . 861 . 872 . 887 . 899 . 911 . 921 . 930 . 937	0. 748 . 799 . 824 . 845 . 862 . 875 . 886 . 905 . 919 . 933 . 944 . 963 . 962		

TABLE II.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA—Continued VALUES OF  $L_a$  FOR TAPERED WINGS WITH ROUNDED TIPS,  $c_{l_a i} = \frac{S}{cb} L_a$ 

c <sub>i</sub>  c <sub>s</sub>	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0			
				sr	ANWISI	STATI	on #=	0. 9						
2	0. 378 . 352 . 331 . 314 . 300 . 290 . 282 . 266 . 253 . 245 . 239 . 234 . 231	0. 465 . 447 . 435 . 424 . 416 . 410 . 403 . 383 . 376 . 370 . 366 . 367 . 368	0.508 -500 -495 -490 -487 -484 -481 -472 -469 -468 -468 -470 -473	0. 525 . 528 . 532 . 531 . 531 . 535 . 536 . 541 . 542 . 545 . 547 . 552 . 550	0. 531 . 543 . 554 . 560 . 565 . 572 . 579 . 590 . 597 . 602 . 609 . 618 . 625	0. 534 . 552 . 569 . 583 . 603 . 612 . 628 . 639 . 648 . 659 . 669 . 679	0. 535 . 559 . 581 . 600 . 615 . 628 . 638 . 656 . 649 . 684 . 698 . 710 . 722	0. 536 . 564 . 590 . 613 . 631 . 646 . 653 . 679 . 698 . 715 . 729 . 743 . 759	0. 537 . 568 . 598 . 623 . 643 . 660 . 673 . 698 . 718 . 739 . 756 . 773 . 791	0. 538 . 571 . 603 . 630 . 652 . 671 . 686 . 712 . 736 . 759 . 780 . 800 . 819	0. 539 . 575 . 609 . 636 . 659 . 678 . 696 . 723 . 751 . 776 . 801 . 822 . 846			
		SPANWISE STATION $\frac{y}{5/2}$ =0.95												
2 3	0. 231 . 209 . 191 . 176 . 166 . 155 . 148 . 138 . 132 . 129 . 126 . 122 . 121	0. 296 . 290 . 286 . 281 . 278 . 272 . 261 . 255 . 254 . 252 . 252 . 254 . 252 . 254 . 258	0. 334 . 339 . 342 . 344 . 346 . 346 . 346 . 346 . 348 . 349 . 351 . 357 . 364	0. 358 .369 .378 .384 .392 .393 .403 .410 .419 .423 .432 .432 .439	0. 370 .389 .402 .415 .428 .438 .446 .460 .473 .482 .495 .503 .516	0. 379 . 401 . 420 . 438 . 451 . 464 . 475 . 495 . 511 . 529 . 546 . 558 . 569	0. 381 -407 -428 -449 -466 -481 -495 -520 -542 -562 -581 -598 -613	0. 383 .412 .434 .458 .475 .494 .510 .538 .588 .610 .629 .648	0. 386 416 440 463 482 502 521 553 583 609 635 658	0. 388 - 418 - 444 - 469 - 510 - 529 - 566 - 598 - 628 - 655 - 682 - 707	0. 390 420 446 471 496 515 534 575 608 640 671 702 730			
				sı	PANWISI	S STATI	ON #=	0. 975		_				
2 3 4 5 6 7 8 10 12 14 16 18	0. 132 .119 .107 .098 .089 .081 .077 .069 .063 .064 .063 .062	0. 172 . 166 . 163 . 158 . 158 . 158 . 158 . 161 . 163 . 166 . 169 . 171	0. 207 . 210 . 214 . 217 . 219 . 222 . 228 . 233 . 242 . 248 . 255 . 263 . 271	0. 239 . 250 . 288 . 269 . 272 . 278 . 283 . 295 . 303 . 320 . 331 . 346 . 363	0. 263 .278 .288 .304 .314 .320 .328 .343 .360 .376 .394 .412 .435	0. 272 .289 .304 .320 .352 .362 .373 .395 .413 .435 .461	0, 274 .291 .308 .322 .340 .351 .303 .390 .413 .403 .492 .515	0. 277	0. 279 .298 .315 .333 .380 .366 .383 .415 .448 .478 .510 .539 .570	0. 281 . 300 . 319 . 338 . 357 . 373 . 391 . 428 . 461 . 495 . 529 . 560 . 593	0. 282 .301 .322 .342 .361 .481 .400 .438 .473 .510 .546 .580 .615			

TABLE III.—ADDITIONAL SPAN LIFT-DISTRIBUTION DATA FOR THE ELLIPTICAL WING,  $c_{l_{a1}} = \frac{S}{cb} L_a$ 

<u>₹</u>	$L_{\bullet}$
0 .2 .4 .6 .8 .9 .95	1. 273 1. 248 1. 167 1. 019 . 764 . 555 . 398 . 283

TABLE IV.—CALCULATION OF LIFT DISTRIBUTION FOR ILLUSTRATIVE EXAMPLE

<u>y</u> <u>b/2</u>	c	αο	L,	$L_{\epsilon}$	1c13	2c1 a1	$C_L \times c_{l_{a1}}$	c <sub>l</sub>	l <sub>b</sub>	l <sub>a</sub>	ı	C <sub>me-e</sub> .	c <sub>ma.s.</sub> ×c²
0 .2 .4 .6 .8 .9 .95 .975	9. 13 8. 23 7. 30 6. 39 5. 42 4. 49 3. 43 2. 47	0.097 .097 .098 .098 .099 .099 .099 .099	-0. 252 176 018 . 101 . 160 . 159 . 128 . 088	1.300 1.236 1.138 .993 .775 .595 .451 .332	0.127 .098 .012 073 138 165 175 167	0. 950 1. 003 1. 039 1. 036 . 954 . 884 . 877 . 896	1. 140 1. 205 1. 248 1. 242 1. 145 1. 061 1. 053 1. 076	1. 267 1. 303 1. 260 1. 169 1. 007 . 896 . 878 . 909	11. 59 8. 05 88 -4. 66 -7. 48 -7. 41 -6. 01 -4. 13 0	104. 0 99. 0 91. 0 79. 6 62. 0 47. 7 36. 2 26. 6	115.6 107.2 92.0 74.7 54.6 40.3 30.1 22.4	-0.083 075 067 060 052 048 046 045 (044)	-6.92 -5.06 -3.57 -2.45 -1.53 97 54 27

 $<sup>1</sup>c_{l_b} = \frac{\epsilon a_0 S}{cb} L_b = -47.3 \frac{a_0}{c} L_b$ 

 $<sup>{}^{2}</sup>c_{l_{q1}} = \frac{S}{ch}L_{q} = \frac{6.67}{c}L_{q}$ 

TABLE V.—SUMMARY OF TEST RESULTS

[Effective Reynolds number, approximately 8,000,000]

Wing 1	C <sub>Lmax</sub>	C <sub>Demin</sub>	$C_{L_{\max}} / C_{D_{\theta_{\min}}}$	<b>2 p</b> S/b	² <u>h</u> S  b	C
00-0-0 24-0-0 24-15-0 24-30-0 24-30-8-50 21: -18-8-50 21: -15-0 00-16-3-45 00-15-3-45(4-1)	1.53 1.63 1.43 1.51 1.59 1.50 1.48 1.32	0.0076 .0077 .0076 .0076 .0084 .0092 .0078 .0081	201 218 215 188 180 173 192 183 161	0.320 .312 .685 1.108 1.119 .681 .679	0.047 .051 .051 .084 .040 .055 .084 .068	0 040 043 042 - 002 - 003 - 004 - 007 - 005

<sup>&</sup>lt;sup>1</sup> The first group of numbers designates the mean line of the airfoll sections; the next group gives the angle of sweepback in degrees; the last group gives the angle of washout in degrees.

<sup>2</sup> Coordinates of the aerodynamic center: p is the distance from the leading edge of the root chord; and h is the distance above the root chord.

TABLE VI.—COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES

Wing	C	7_0	<u>x.</u>	<del>ij</del> 6	α,	(L-0)	а	
Wing	Calcu- lated	Experi- mental	Calcu- lated	Experi- mental	Calcu- lated	Experi- mental	Calcu- lated	Experi- mental
00-00	0 043 043 043 010 . 006 . 004 . 010	0 040 043 042 .003 .003 .004 .007	0 0 .345 .744 .744 .345 .345 .345	-0.014 022 .352 .775 .786 .348 .351 .346 .334	0 -1.7 -1.7 -1.7 -1.7 6 1.1	0 -1.7 -1.9 -1.9 -1.7 1.2 7	0. 074 . 074 . 074 . 074 . 074 . 074 . 074 . 074 . 075	0.075 .074 .075 .072 .070 .070 .078 .076